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A REVIEW OF FAILURE MODELS FOR CERAMIC MATRIX COMPOSITE LAMINATES UNDER MONOTONIC LOADS

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ABSTRACT

Ceramic matrix composites offer significant potential for improving the performance of turbine engines. In order to achieve their potential, however, improvements in design methodology are needed. In the past most components using structural ceramic matrix composites were designed by "trial and error" since the emphasis on feasibility demonstration minimized the development of mathematical models. To understand the key parameters controlling response and the mechanics of failure, the development of structural failure models is required. A review of short term failure models with potential for ceramic matrix composite laminates under monotonic loads is presented. Phenomenological, semi-empirical, shear-lag, fracture mechanics, damage mechanics, and statistical models for the fast fracture analysis of continuous fiber unidirectional ceramic matrix composites under monotonic loads are surveyed.

INTRODUCTION

Ceramic matrix composites offer significant potential for improving the thrust-to-weight ratio of gas turbine engines by enabling higher cycle temperatures with the use of refractory, high specific strength material systems. Adding a reinforcing or toughening second phase with optimal interfacial bonding improves fracture toughness and decreases the sensitivity of the brittle matrix to pre-existing flaws. The reinforcing second phase can have a variety of shapes, ranging from nearly spherical particles, through whiskers and chopped fibers with various length-to-diameter ratios, to continuous fibers. Aveston et al. (1971) have shown, however, that the addition of continuous small diameter fibers reinforce ceramics most efficiently since their orientation in the direction of the principal load significantly enhances the matrix cracking strain, as well as the ultimate load carrying capability of the composite.

The primary purpose of this increase in toughness is to allow for a "graceful" rather than catastrophic failure as opposed to an increase in the ultimate strength, although in high fiber volume fraction composites, that may also occur. Ceramic matrix composites retain substantial strength and strain capability beyond the initiation of first matrix cracking despite the fact that neither of its constituents would exhibit such behavior if tested alone. First matrix cracking occurs at a strain greater than that in the monolithic ceramic alone. As additional load is applied beyond first matrix cracking, the matrix will break in a series of transverse cracks bridged by reinforcing fibers. Additional load is born by the fibers until the ultimate strength of the composite is reached. The desired design stress limit, however, should be less than the matrix cracking stress as cracking allows oxidation of the fibers, especially at elevated temperatures, which causes increased fiber-matrix bonding and embrittlement of the composite

In the past, most components using structural ceramic matrix composites were designed by "trial and error," since the emphasis was on feasibility demonstration rather than on fully understanding the parameters controlling behavior In addition, the continuous change and development of these material systems and the lack of standardized design data minimized the emphasis on mathematical modeling. To gain insight into the mechanisms of failure, and to understand the parameters controlling response the development of structural failure models is required.

The objective of this survey is to investigate appropriate failure models which may be applicable to the fast fracture analysis of continuous fiber unidirectional ceramic matrix composite lamina under monotonic loading, both for first matrix cracking and ultimate strength. Much of this methodology has been adapted from existing polymer matrix composites technology. Phenomenological, semi-empirical, shear-lag, fracture mechanics, damage mechanics and statistical models are surveyed. Though semi-empirical models apply to multidirectional laminates they are included here for completeness. The emphasis is not on evaluating the models in detail; more complete surveys are available elsewhere. Rather the ability of the models to predict the fast fracture of ceramic matrix composites is discussed.

Future work will selectively implement these models and others to be developed into an integrated composite

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design code for the reliability analysis of ceramic matrix composites for use by industry in designing heat engine components. Because a general purpose code is desired, failure criteria applicable to unidirectional lamina are preferred. Laminate failure criteria would require the characterization of each laminate configuration under consideration. Classical lamination theory can be used to determine the failure of the laminate from the failure of the individual lamina. The authors are aware (Labossiere and Neale, 1987) that variation of the stacking sequence will affect the strength of a laminate, while classical lamination theory predicts the same strength if the layers in a symmetric laminate are rearranged into another symmetric laminate. This issue will also be addressed in the future.

PHENOMENOLOGICAL FAILURE CRITERIA

A number of theories exist to predict the failure of homogeneous isotropic materials under general states of stress using the material properties obtained from simple uniaxial tension, compression and shear tests. Generalizations of these criteria for homogeneous, isotropic materials have been proposed as failure criteria for fiber reinforced composites that are anisotropic and inhomogeneous. These phenomenological criteria are the most familiar to design engineers. Consequently, models such as maximum stress, maximum strain, Azzi-Tsai (Azzi and Tsai, 1965), and Tsai-Wu (Tsai and Wu, 1971) are currently the most frequently used in industry to design polymer matrix composite components according to a user survey by Burk (1983) and Soni (1983).

Two excellent reviews of phenomenological failure criteria are by Nahas (1986) and Labossiere and Neale (1987). Other surveys of phenomenological anisotropic failure theories exist (Sandhu, 1972; Bert et al., 1969; Kaminski and Lantz, 1969; Sendeckyj, 1972; Vicario and Toland, 1975; Rowlands, 1975; Tsai, 1984; Snell, 1978). Nahas, in his review, classifies failure criteria into four categories: direct laminate criteria, limit criteria, interaction criteria and tensor polynomial criteria. In the direct laminate approach the failure criterion is applied to the entire laminate which is considered homogeneous but anisotropic. This requires the strength characterization of each laminate under consideration; thus, these criteria are not amenable to a general purpose code as discussed earlier and are not considered further.

In the ply-by-ply approach, which the other three categories fall into, the lamina is considered to be homogeneous and orthotropic. Lamination theory is used to find the stresses in each lamina and these stresses are transformed to the lamina principal material axes before the failure criteria are applied. The direction of the principal stresses or strains have no significance for isotropic materials. However, strength varies with direction in composites, and generally the direction of maximum strength does not necessarily coincide with the direction of principal stress. Thus, the highest stress may not be the stress governing the design and a comparison of the actual stress field with the allowable stress field is required. The allowable stress field for a unidirectional composite is given by the strengths in the principal material direction. They are the longitudinal tensile strength, longitudinal compressive strength, transverse tensile strength, transverse compressive strength, and in-plane shear strength.

Limit criteria, the first category in the ply-by-ply approach, assume that failure occurs when the stress in one of the principal material directions exceeds the allowable value. The maximum stress criterion and the maximum strain criterion are examples of limit criteria, that is

Maximum stress

$$X' \le \sigma_1 \le X\,, \qquad Y' \le \sigma_2 \le Y\,, \qquad |\tau_{12}| \le S$$

Maximum strain

$$\varepsilon'_{1U} \le \varepsilon_1 \le \varepsilon_{1U}\,, \qquad \varepsilon'_{2U} \le \varepsilon_2 \le \varepsilon_{2U}\,, \qquad |\gamma_{12}| \le \gamma_{12U} \tag{1}$$

where σ and τ are the stresses, ε and γ are the strains, X, Y and S are the longitudinal, transverse and shear strengths, prime (') denotes compressive strength, subscripts 1 and 2 denote longitudinal and transverse directions and U is the ultimate strength or strain.

Stowell and Liu (1961), Kelly and Davies (1965), and Prager (1969) suggested additional variations of the maximum stress criterion. Hu (1985), Wasti (1970), and Lance and Robinson (1971) have proposed generalizations of Tresca's maximum shear stress criterion for anisotropic materials.

Hill (1948) proposed an interactive criterion by generalizing the Von Mises Hencky maximum distortional energy theory to include anisotropy in metals such as cold rolled steel. It is assumed that the yield condition is a quadratic function of the stress components,

$$2f(\sigma_{ij}) \equiv F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L\tau_{23}^2 + 2M\tau_{31}^2 + 2N\tau_{12}^2 = 1 \tag{2}$$

where F, G, H, L, M, and N are material coefficients characteristic of the state of anisotropy, and the subscripts i and j refer to the material axes. Linear terms were not included since it was assumed that there is no Bauschinger effect. Hill showed that the material coefficients are functions of the material characteristic strengths, and that for plane stress the criterion reduces to

$$\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma_1\sigma_2 + \left(\frac{\tau_{12}}{S}\right)^2 = 1\,. \tag{3}$$

Azzi and Tsai (1965) suggested that Hill's criterion be modified for composites by assuming that the material is transversely isotropic and setting Y = Z.

$$\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 - \left(\frac{\sigma_1\sigma_2}{X^2}\right) + \left(\frac{\tau_{12}}{S}\right)^2 = 1\,. \tag{4}$$

This criterion is also referred to as the Tsai-Hill or maximum work criterion.

Hoffman (1967) modified Hill's criterion to take into account the differences between tensile and compressive strengths. Other interactive criteria similar to Azzi-Tsai have been proposed by Marin (1957), Franklin (1968), Norris and McKinnon (1946), Yamada and Sun (1978), Fischer (1967), Chamis (1969), and Griffith and Baldwin (1962). For various reasons they are not as widely used or accepted

In the 1960's, the use of tensor polynomial criteria was motivated by the idea that the failure envelopes of fiber reinforced materials should be invariant with respect to the choice of material axes. With that in mind Gol'denblat and Kopnov (1966) proposed the first tensor polynomial criterion for anisotropic materials, that is

$$\left(F_i\sigma_i\right)^{\kappa} + \left(F_{ij}\sigma_i\sigma_j\right)^{\lambda} + \left(F_{ijk}\sigma_i\sigma_j\sigma_k\right)^{\mu} + \ldots = 1 \qquad i,j = 1,6 \qquad (5)$$

where the F's denote strength tensors of various orders and κ, λ and μ are experimentally determined constants.

Tsai and Wu (1971) later proposed a similar criterion, that is

$$f(\sigma) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \qquad i,j = 1,6 \qquad (6)$$

To ensure that the failure criterion is physically meaningful, or that the failure surface is closed, the stability criterion, $F_{ii}F_{jj} - F_{ij}^2 > 0$ (no summation), must be satisfied. Under plane stress conditions the criterion reduces to

$$F_1\sigma_1 + F_2\sigma_2 + F_6\tau_{12} + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\tau_{12}^2 = 1 \qquad (7)$$

which is the Hill criterion with the addition of linear terms. The main disadvantage of the Tsai-Wu criterion is that F_{12} is determined from difficult biaxial tests. This theory has not been as widely accepted because of disagreements over the experimental methods to determine F_{12}. According to Narayanaswami and Adelman (1977), in the off-axis tests of four different fiber orientations, it was found that only one value satisfied the necessary stability criterion.

Wu (1974) claims that most phenomenological criteria are a degenerate case of the Tsai-Wu criterion. Ashkenazzi (1966), Malmeister (1967), and Huang and Kirmser (1975) have also proposed tensor polynomial failure criteria, however, they have only been used in special cases which reduce them to the Tsai-Wu theory.

SEMI-EMPIRICAL FAILURE CRITERIA

It is natural to attempt to apply linear elastic fracture mechanics (LEFM) to tension loaded composites with crack-like defects, however, according to Wu (1968) LEFM is valid only if:

(1) The orientation of the flaws with respect to the principal axes of symmetry is fixed.

(2) The stress intensity factor defined for anisotropic cases is consistent with the isotropic case in stress distribution and in crack displacement modes; and

(3) The critical orientation coincides with one of the principal directions of elastic symmetry.

In general, however, significant amounts of damage growth at the crack tip precede fracture in a composite and self-similar crack growth is not likely to occur, even for unidirectional laminates.

To apply principles of LEFM to composites, the previously proposed fracture theories of Waddoups et al. (1971), Cruse (1973), Whitney and Nuismer (1974), and Nuismer and Whitney (1975) did not consider this complex pattern of crack tip damage. Instead the damage was modeled as an "intense energy" region that was assumed to grow in a self-similar manner. With these assumptions fracture mechanics models developed for isotropic materials were generalized for composites.

Consider the two fracture models proposed by Waddoups, Eisenmann, and Kaminski (WEK)(1971), one for laminates containing circular holes and one for laminates containing straight cracks. For circular holes, the WEK model assumes that regions of intense energy of length "a" transverse to the loading direction are developed at the edge of the hole, as shown in Fig. 1. From Bowie (1956) and Paris and Sih (1965) they solve for the opening mode stress intensity factor, K_I

$$K_I = \sigma_N^{\infty}\sqrt{\pi a}\, f\left(\frac{a}{R}\right) \qquad (8)$$

where R is the radius of the hole, a is the characteristic length of the intense energy region, and σ_N^{∞} is the remote applied stress. Values of f(a/R) can be found in Paris and Sih (1965). At failure the notched strength of the composite laminate, σ_N^{∞}, from Eq. (8) is

$$\sigma_N^{\infty} = \frac{K_{IC}}{\sqrt{\pi a}\, f(a/R)} \qquad (9)$$

where K_{IC} is the composite critical stress intensity factor or toughness. For an unnotched specimen, that is for a composite with no hole,

$$\sigma_o = \sigma_N^{\infty}\Big|_{a/R=\infty} = \frac{K_{IC}}{\sqrt{\pi a}\ (1.00)} \qquad (10)$$

Combining Eqs. (9) and (10), the notched strength of the composite laminate is

$$\sigma_N^{\infty} = \frac{\sigma_o}{f(a/R)} \qquad (11)$$

The WEK fracture model for straight cracks assumes that the half-crack length, c, is extended by the damage zone, a. This results in an effective half-crack length of (c + a), similar to Irwin's plastic zone correction factor for isotropic materials. The resulting strength of the notched composite laminate is

$$\sigma_N^{\infty} = \sigma_o\sqrt{\frac{a}{c+a}} \qquad (12)$$

Other similar semi-empirical models have been proposed by Whitney and Nuismer (1974), Nuismer and Whitney (1975), Karlak (1977), Pipes et al. (1979), Pipes et al. (1980), Mar and Lin (1977), Poe and Sova (1980), Tan (1987), and Zhen (1983). A comprehensive review is given by Awerbuch and Madhukar (1985). Two excellent discussions of semi-empirical models are given by Kanninen et al. (1977), and Kanninen and Popelar (1985).

SHEAR-LAG FAILURE CRITERIA

Hedgepeth (1961) was the first to apply shear-lag models to unidirectional composites. The shear-lag models assume that the load is transferred from broken fibers to adjacent fibers by the matrix shear forces which are assumed to be independent of the transverse displacements. This uncouples the longitudinal equilibrium equations from those in the transverse direction.

As shown in Fig. 2, Hedgepeth's model consists of a sheet of parallel filaments which carry axial loads only in a matrix which carries only shear. Equilibrium of an element of the n^{th} filament, for the static case, requires that

$$\frac{dp_n(x)}{dx} + s_n(x) - s_{n-1}(x) = 0 \qquad (13)$$

where $p_n(x)$ is the load in the n^{th} filament and $s_n(x)$ is the matrix shear force per unit length between the n^{th} and $(n+1)^{th}$ filaments. The force in the n^{th} filament is

$$p_n(x) = EA \frac{du_n(x)}{dx} \tag{14}$$

where EA is the extensional stiffness and $u_n(x)$ is the displacement of the n^{th} filament in the axial direction. The shear force per unit length can be calculated from

$$s_n(x) = \frac{Gh}{d}\left(u_{n+1} - u_n\right) \tag{15}$$

where G is the matrix shear modulus, h is the effective thickness of the matrix, and d is the centerline spacing between the filaments. Equation (15) is the shear-lag assumption which assumes that the shear force is a function of the axial displacements only.

Substitution of Eqs. (14) and (15) into Eq. (13) results in

$$EA \frac{d^2u_n}{dx^2} + \frac{Gh}{d}\left(u_{n+1} - 2u_n + u_{n-1}\right) = 0 \tag{16}$$

Applying the boundary conditions for "n" broken fibers, Hedgepeth solved for the stress concentration in the first unbroken fiber as a function of the number of broken fibers.

Hedgepeth and Van Dyke (1967) extended the analysis to consider three-dimensional fiber arrays and an elastic-perfectly plastic matrix. Eringen and Kim (1974) further generalized Hedgepeth's model to include transverse loads in the matrix. Both of these models, however, only determined stress concentrations in unbroken fibers. Goree and Gross (1979) investigated the behavior due to broken fibers and matrix damage in the form of longitudinal yielding and matrix splitting, as shown in Fig. 3. A failure criterion for the matrix due to shear alone was assumed in attempting to predict the characteristic strength and fracture properties of unidirectional composite lamina. Further models by Dharani et al. (1983), and Kaw and Goree (1985) considered other forms of damage.

FRACTURE MECHANICS FAILURE CRITERIA FOR FIRST MATRIX CRACKING

According to Aveston, Cooper, and Kelly (ACK) (1971), if the fibers have a higher failure strain than the matrix, multiple fracture of the matrix will occur if

$$\sigma_{fu}V_f > \sigma_{mu}V_m + \sigma'_fV_f \tag{17}$$

where V_f and V_m are the fiber and matrix volume fractions, σ_{fu} and σ_{mu} are the ultimate strength of the fiber and matrix and σ'_f is the stress on the fibers required to produce a strain equal to the failure strain of the matrix. Most ceramic matrix composites exhibit such multiple matrix cracking behavior where a crack propagates through the matrix and is bridged by reinforcing fibers. ACK assumed that the requirements necessary for the formation of such a matrix crack are that the stress in the matrix must be equal to the matrix breaking stress, and that the total energy of the specimen and the loading system must decrease as a result of crack formation.

The energy changes considered by ACK when a matrix crack is formed are: ΔW, the work done by the applied stress; γ_{db}, the work done in breaking the fiber-matrix interfacial bond; ΔU_m, the decrease in the matrix strain energy; ΔU_f, the increase in fiber strain energy; and U_s, the work done against frictional forces as the fiber moves relative to the matrix. The condition for the formation of a crack then is

$$2\gamma_mV_m + \gamma_{db} + U_s + \Delta U_f \leq \Delta W + \Delta U_m \tag{18}$$

where γ_m is the fracture surface work in forming a matrix crack. The energy terms can be calculated from

$$\begin{aligned}
\Delta W &= E_fE_mV_m\varepsilon_{mu}^3 \frac{\alpha r(1+\alpha)}{2\tau_s} \\
\Delta U_m &= E_fE_mV_m\varepsilon_{mu}^3 \frac{\alpha r}{3\tau_s} \\
\Delta U_f &= E_fE_mV_m\varepsilon_{mu}^3 \frac{\alpha r(1+\alpha/3)}{2\tau_s} \\
U_s &= E_fE_mV_m\varepsilon_{mu}^3 \frac{\alpha r(1+\alpha)}{6\tau_s} \\
\gamma_{db} &= 2\sigma_{mu}V_m \frac{G_{II}}{\tau_s}
\end{aligned} \tag{19}$$

where E_f and E_m are Young's modulus for the fiber and matrix, ε_{mu} is the failure strain of the matrix, τ_s is the fiber-matrix interfacial shear strength, G_{II} is the energy per unit area required to debond the fiber from the matrix, r is the fiber radius, and $\alpha = E_mV_m/E_fV_f$. For a purely frictional bond γ_{db} equals zero and the critical matrix cracking strain can be expressed as

$$\varepsilon_{mu} = \left(\frac{12\tau_s\gamma_mE_fV_f^2}{E_c^2E_m^2rV_m}\right)^{1/3} \tag{20}$$

where E_c is Young's modulus of the composite. Aveston and Kelly (AK) (1975) later extended the analysis to include an elastically bonded interface and an initially bonded, debonding interface, which are shown in Fig. 4 along with the unbonded, slipping fibers case.

ACK looked at the energy states before and after crack propagation, while Budiansky, Hutchinson and Evans (BHE) (1986) considered the case of steady state cracking which assumes that the stress necessary to propagate a crack larger than some characteristic dimension is constant. The assumption of steady-state cracking implies that the stress at the crack tip remains unchanged during crack growth and also that the upstream and downstream stresses, far ahead of and behind the crack, do not change.

BHE considered a body in three different states which are shown in Fig. 5. In state (0) the body has no external applied loads but contains an initial stress distribution, σ_0. In state (1), with an external applied load T, the internal stress distribution becomes σ_1 and additional strains, ε_1 are produced. Obviously, these subscripts do not correspond to the previously introduced material axes directions. Open cracks as well as internal surfaces in which sliding has occurred may exist. In state (2) additional cracking and frictional sliding has occurred with no change in T. The stresses and strains are σ_2 and ε_2, respectively. The loss in potential energy in going from state (1) to state (2) may be written as

$$\pi_1 - \pi_2 = \frac{1}{2}\int_V(\sigma_1 - \sigma_2):(\varepsilon_1 - \varepsilon_2)dV + \xi_f \tag{21}$$

where π_1 and π_2 are the potential energies in states (1) and (2) and ξ_f is the frictional energy.

If no slipping occurs, Fig. 4(a), or if the fibers are slipping but frictionally constrained, Fig. 4(b), the potential energy release rate per unit crack extension per unit thickness is

$$P_u - P_d = \frac{1}{2A_c}\int_{-L}^{L}\int_{A_c}(\sigma_u - \sigma_d):(\varepsilon_u - \varepsilon_d)dA\,dz + \frac{\partial \varepsilon_f}{\partial s} \tag{22}$$

where P_u and P_d are the upstream and downstream potential energies per unit cross sectional area of composite crack extension, σ_u, ε_u, σ_d, and ε_d are the upstream and downstream stress and strain distributions and A_c is the representative cross section of the composite crack extension. The frictional energy dissipation rate associated with fiber-matrix slip is $\partial\varepsilon_f/\partial s$.

BHE assume that the energy release rate, $P_u - P_d$, must be balanced by the sum of the frictional energy dissipation rate and the critical matrix crack extension energy release rate, $V_m G_m$, where G_m is the critical matrix energy release rate. Thus,

$$\frac{1}{2}A_c\int_{-L}^{L}\int_{A_c}(\sigma_u - \sigma_d):(\varepsilon_u - \varepsilon_d)dA\,dz = V_m G_m \tag{23}$$

In the case of initially bonded, debonding fibers a term for the debonding energy release rate is included in the materials resistance to crack growth,

$$\frac{1}{2}A_c\int_{-L}^{L}\int_{A_c}(\sigma_u - \sigma_d):(\varepsilon_u - \varepsilon_d)dA\,dz = V_m G_m + 4V_f\left(\frac{l_d}{r}\right)G_d \tag{24}$$

where l_d is the debond length, r is the fiber radius and G_d is the critical debonding energy release rate.

The upstream stresses are given by the rule-of-mixtures relationship,

$$\sigma_f^u = \left(\frac{E_f}{E_c}\right)\sigma + \sigma_f^I, \quad \text{and} \quad \sigma_m^u = \left(\frac{E_m}{E_c}\right)\sigma + \sigma_m^I \tag{25}$$

where σ_f^I and σ_m^I are the initial residual axial fiber and matrix stresses in the unloaded composite, and σ is the average applied stress. The downstream stresses are determined from a shear-lag analysis of a composite cylinder. Substitution of the upstream and downstream stresses into Eqs. (23) and (24) results in the matrix cracking condition. For unbonded frictionally constrained fibers the matrix cracking stress, σ_{cr}, is

$$\frac{\sigma_{cr}}{E_c} + \frac{\sigma_m^I}{E_m} = \left(\frac{V_f E_f G_m \rho}{r E_m E_c}\right)^{1/2} \tag{26}$$

where r is the fiber radius and ρ is defined in BHE as a function of constituent properties. The matrix cracking stress for unbonded slipping fibers is

$$\frac{\sigma_{cr}}{E_c} + \frac{\sigma_m^I}{E_m} = \left(\frac{6V_f^2 E_f \tau_s}{V_m E_m E_c}\right)^{1/3}\left(\frac{G_m}{r E_m}\right)^{1/3} \tag{27}$$

where τ_s is the fiber-matrix interfacial shear stress. Equations (26) and (27) are identical to the AK/ACK results except for the initial matrix stresses. BHE also derive a parametric relationship for the results between the no-slip and large slip cases, and the matrix cracking stress for the case of initially bonded, debonding fibers. These cases were not considered by AK/ACK.

Marshall, Cox, and Evans (MCE) (1985) defined steady state cracking differently, distinguishing between large and small cracks, as shown in Fig. 6. Large cracks asymptotically approach the equilibrium separation, u_o, of the completely failed matrix bridged by reinforcing fibers. This equilibrium separation occurs a characteristic dimension, c_o, from the crack tip. Beyond this characteristic distance the net force in the fibers that bridge the crack exactly balances the applied force and the stress needed to extend the crack is independent of the crack length. Crack growth in this region is defined by MCE as steady state growth. On the other hand, for short cracks the entire crack contributes to the stress concentration, and the stress required to propagate a crack is sensitive to crack length.

Unlike BHE, MCE consider unbonded, frictionally constrained fibers only. The matrix cracking stress is evaluated using a stress intensity approach. The fibers and matrix are assumed to be cut, as shown in Fig. 7(a), and traction forces, T, are applied to the ends of the fibers so that they are rejoined. The crack surfaces are regarded as being subjected to a net opening pressure $[\sigma_\infty - p(x)]$ where σ_∞ is the applied load and $p(x)$ is the distribution of closure pressure on the crack surfaces defined as

$$p(x) = T(x)V_f \tag{28}$$

where x represents the position on the crack surface, as shown in Fig. 7(b). Therefore, MCE introduce the composite stress intensity factor

$$K^L = 2\left(\frac{c}{\pi}\right)^{1/2}\int_0^1 \frac{[\sigma_\infty - p(X)]}{\sqrt{1 - X^2}}\,dX \tag{29}$$

for a straight crack or

$$K^L = 2\left(\frac{c}{\pi}\right)^{1/2}\int_0^1 \frac{[\sigma_\infty - p(X)]}{\sqrt{1 - X^2}}X\,dX \tag{30}$$

for a penny shaped crack, where c is the crack length and $X = x/c$. MCE assume that the critical matrix stress intensity factor is related to the composite stress intensity factor by

$$K_c^m = \frac{K_c^L E_m}{E_c} \tag{31}$$

The steady-state matrix cracking stress is obtained by evaluating Eqs. (29) or (30) and (31) and setting

$\sigma_\infty = \sigma_{cr}$. After performing the indicated operations one obtains

$$\sigma_{cr} = \delta' \left[\left(1 - \nu^2\right) K_c^{m2} \tau_s E_f V_f^2 V_m \frac{(1+\eta)^2}{E_m r} \right]^{1/3} \quad (32)$$

where ν is Poisson's ratio, τ_s is the fiber-matrix interfacial shear stress, $\eta = E_f V_f / E_m V_m$, r is the fiber radius and δ' is a dimensionless constant. The results are equivalent to those for the large slip case of ACK, where $\delta' = 1.83$ from an MCE analysis and $\delta = 6^{1/3}$ from an ACK analysis.

In subsequent papers Marshall and Cox (1987) generalized the analysis to include fiber failure in the wake of the crack, as shown in Fig. 8(a). In their model the fibers were assumed to have a singled valued strength. In another model, Thouless and Evans (1988) assumed a statistical variation in the fiber strength, which is also shown in Fig. 8(b), and examined the effects of pull-out when fibers fracture away from the crack plane.

CONTINUUM DAMAGE MECHANICS

Continuum damage mechanics will now be discussed because it addresses local phenomena as did fracture mechanics previously and because it can be utilized to predict the failure of composites, especially time dependent failure. However, it will only be briefly described here in general terms and will not be included in the subsequent discussion.

The basic idea of damage mechanics according to Sidoroff (1984) is the introduction of a damage variable describing at the macroscopic level the microscopic degradation occurring in the material. Like fracture mechanics, continuum damage mechanics considers the behavior of an imperfect body, however, while fracture mechanics deals with one dominant macrodefect, continuum damage mechanics considers a whole population of microdefects. In other words, where fracture mechanics starts with a cracked body, continuum damage mechanics starts with a perfect material and follows its damage accumulation to the appearance of a dominant macroscopic defect.

In the simplest case a single damage variable D is assumed to predict failure when $D = 1$ or some other critical value D_c and it is usually defined from some measurable macroscopic quantity ρ which is hoped to be representative of the macroscopic degradation process. Hence,

$$D = \frac{\rho_0 - \rho}{\rho_0 - \rho_R} \quad (33)$$

where ρ_0 and ρ_R denote the values of ρ in the initial undamaged state and at rupture. To complete the model a damage evolution law is needed to follow the damage process. A generalized evolution equation can be written in the form

$$\frac{dD}{dt} = H(\sigma, \dot{\sigma}, D \ldots) \quad (34)$$

This one-dimensional scalar model, however, will not be sufficient for composite structures. Many mechanisms are known to be responsible for composite damage, i.e., fiber failure, matrix cracking, interfacial debonding, etc. and their interaction is not clearly understood. Averaging them in a single damage variable cannot be realistic. They are also strongly related to the anisotropy of composite materials. Consequently, anisotropic damage tensors and other damage descriptions have been proposed by Wnuk and Kriz (1985), Talreja (1986), Shen et al. (1987), Lene (1986), and others.

STATISTICAL FAILURE CRITERIA

Purely statistical criteria have also been proposed for the failure characterization of ceramic matrix composites when the deterministic models do not adequately describe observed variation in composite strength. Because these criteria are purely statistical they can always be applied, but they do require a greater amount of test data.

The principle of independent action (PIA) (Wetherhold, 1983) is the statistical formulation of the maximum stress criterion. Using Weibull's weakest link statistic and a state of plane stress for a transversely isotropic material, the reliability due to intrinsic flaws in a composite can be calculated from

$$R = \exp \int_V - \left[\left(\frac{\sigma_1}{\beta_1}\right)^{\alpha_1} + \left(\frac{\sigma_2}{\beta_2}\right)^{\alpha_2} + \left(\frac{\sigma_6}{\beta_6}\right)^{\alpha_6} \right] dV \quad (35)$$

where the reliability, R, is the probability of no failure of the volume, α_i and β_i are the Weibull shape and scale parameters and σ_i are the in-plane material axes stresses and σ_6 has been substituted from the previously used τ_{12}. The principle of independent action is a weakest link model which assumes that volume V is divided into N elements where no failure of the volume requires no failure of any of the elements. The event of no failure of an element is the same as the event of no failure of an element by any of the stress components which are assumed to act independently. A statistical equivalent of the maximum strain failure criterion has also been developed.

Wetherhold's other model (Wetherhold, 1983) is the probabilistic form of the maximum distortional energy (MDE) criterion. It also is a weakest link model where MDE is the criterion for failure of an element. The MDE criterion is given by

$$K_{MDE}^2 = \left(\frac{\sigma_1}{X_1}\right)^2 - \frac{\sigma_1 \sigma_2}{X_1^2} + \left(\frac{\sigma_2}{X_2}\right)^2 + \left(\frac{\sigma_6}{X_6}\right)^2 \quad (36)$$

The reliability of the element is $P\{K < 1\}$. To evaluate the reliability one needs to integrate the following equation:

$$R = \int_0^{1(1)} \int_0^{h(1;X_6)} \int_0^{g(1;X_2,X_6)} F_{X_1}(X_1) F_{X_2}(X_2) F_{X_6}(X_6) \cdot dx_1\, dx_2\, dx_6 \quad (37)$$

where F_{X_1}, F_{X_2}, and F_{X_6} are the probability density functions for strengths X_1, X_2, and X_6 and

$$g(1;X_2,X_6) = \sqrt{\frac{\sigma_1^2 - \sigma_1\sigma_2}{1 - (\sigma_2/X_2)^2 - (\sigma_6/X_6)^2}}$$

$$h(1;X_6) = \sqrt{\frac{\sigma_2^2}{1 - (\sigma_6/X_6)^2}}$$

$$f(1) = \sigma_6$$

The integral in Eq. (37) is intractable and can be evaluated only by using a Monte Carlo technique. Other statistical forms of phenomenological criteria are given by Cassenti (1984) and Sun and Yamada (1978).

The previous statistical criteria were based on the observed variation in composite strength. For composites with high fiber volume fractions and fully cracked matrices, three models (Jayatilaka, 1979) for the ultimate unidirectional strength have been proposed that are solely based on the variation in fiber strength of the composite. These models are based on the following assumptions:

(1) The crack propagates catastrophically in a direction normal to the adjacent fibers following the fracture of a single fiber.

(2) The crack may also propagate along the fiber-matrix interface and the composite behaves then as a bundle of unbound fibers.

(3) The last model assumes that when the weakest fiber fails it is followed by additional fiber fracture at other weak sites as the load is increased. This is known as the cumulative damage failure model.

The strength of a single fiber varies with the length of the fiber and this is due to flaws present in the fiber. The average single fiber strength is given by

$$\bar{\sigma}_\ell = \beta(\ell)^{-1/\alpha}\Gamma\left(1 + \frac{1}{\alpha}\right) \tag{38}$$

where α and β are the fiber Weibull shape and scale parameters, ℓ is the fiber length and Γ is the gamma function. The strength of a bundle of fibers in the second model is similar to the problems considered by Daniels (1945) where he studied the strength of a bundle of threads. The average bundle strength of the fibers is

$$\bar{\sigma}_b = \beta(l\alpha e)^{-1/\alpha} \tag{39}$$

where e is the base of the natural logarithm, that is $e = 2.7183$. These strengths are substituted into the rule-of-mixtures relationship to determine the ultimate strength of the composite assuming zero matrix strength.

The cumulative damage model assumes that when a fiber fractures, a portion of the fiber of length δ does not carry any load. The composite is divided into layers of length δ. Each layer is considered a bundle of links and the composite is a series of such bundles, or a chain-of-bundles. Failure of one bundle results in total failure of the composite. The average strength value from the cumulative damage model to be used in the rule-of-mixtures equation is

$$\bar{\sigma}_{fu} = \sigma_{mode} \cong \beta(\alpha\,\delta e)^{-1/\alpha} \tag{40}$$

where δ, the ineffective length, is a function of r, V_f, E_f and the matrix shear modulus.

Other investigators have proposed models based on the cumulative damage model. Zweben and Rosen (1970) considered the chain-of-bundles model in discussing crack growth in unidirectional composites. The strength of each bundle was determined by bundle theory taking into consideration stress concentrations from the shear-lag model of Hedgepeth in fibers adjacent to broken fibers. Weakest link theory was used to determine the probability of failure of the chain-of-bundles. Zweben and Rosen did not succeed in establishing a usable failure criterion.

Harlow and Phoenix (1978) also utilized the chain-of-bundles model. They assumed that the strength of individual brittle fibers, embedded in a matrix having stiffness and strength far below that of the fibers, follows a Weibull distribution. Upon loading isolated fractures are observed in individual fibers. As the load is increased fiber fractures accumulate until the composite can no longer support the load. Two load sharing rules for the unbroken fibers were considered. The Equal Load Sharing (ELS) rule assumes that the load from the broken fibers is evenly distributed amongst the remaining fibers. The Local Load Sharing (LLS) rule assumes that the additional load is concentrated in the fibers adjacent to the broken fibers.

The Batdorf (1982) model, as opposed to the chain-of-bundles models, considers a composite containing N fibers of length ℓ held together by a matrix. Damage resulting from loading is assumed to consist of breaks in the fibers. Single isolated breaks are called singlets, pairs of breaks are called doublets, or in general i-plets. The assumption is made that fiber failure conforms to a Weibull distribution where the cumulative probability of the fiber breaking is given by

$$P_f(\sigma) = 1 - \exp\left[-\ell\left(\frac{\sigma}{\beta}\right)^\alpha\right] \tag{41}$$

where α and β are the Weibull parameters. If there are N fibers of length l, the number of singlets at stress σ is

$$Q_1 = NP_f = N\ell\left(\frac{\sigma}{\beta}\right)^\alpha \tag{42}$$

A singlet becomes a doublet when one of the neighboring fibers breaks. The probability that a singlet becomes a doublet, $P_{1\to2}$, is

$$P_{1\to2} = n_1\lambda_1\left(\frac{c_1\sigma}{\beta}\right)^\alpha \tag{43}$$

where n_1 is the number of fibers subjected to a stress concentration that varies from $c_1\sigma$ to σ relative to the nominal fiber stress and λ_1 is the effective length of this overstressed region. The number of doublets in loading to stress σ thus becomes

$$Q_2 = Q_1n_1\lambda_1\left(\frac{c_1\sigma}{\beta}\right)^\alpha \tag{44}$$

Generalizing this result gives

$$Q_{i+1} = Q_in_i\lambda_1\left(\frac{c_1\sigma}{\beta}\right)^\alpha \tag{45}$$

Failure occurs when an i-plet becomes unstable and immediately becomes an $(i + 1)$ plet, which immediately

becomes an (1 + 2) plet, etc., resulting in fracture of the composite.

SUMMARY AND DISCUSSION

There are two general approaches to predicting failure. One is the phenomenological approach and the other is mechanistic. The phenomenological approach is strictly an empirical curve fitting procedure that develops a surface in stress space to fit the available data. With enough constants the experimental data can always be adequately described. The phenomenological approach has been applied to homogeneous, isotropic materials such as metals with considerable success. If the strength of the metal, however, is sensitive to microstructural discontinuities, such as cracks, a mechanistic approach such as fracture mechanics is required. The mechanistic approach develops criteria that describe the mechanisms of failure in terms of microstructural variables and the use of engineering principles.

The complex mechanisms of failure in ceramic matrix composites such as matrix cracking, interfacial debonding and fiber pull-out (Harris, 1986) are strongly affected by microstructural parameters such as fiber diameter or fiber-matrix interfacial shear strength. The macroscopic phenomenological criteria cannot account for these factors in predicting failure. In addition, mechanistic criteria can extrapolate beyond known test conditions to account for variations in these parameters. Thus, mechanistic criteria are necessary to understand the factors controlling the failure of ceramic matrix composites. On the other hand, they may become intractable when too many parameters control the materials failure behavior.

The semi-empirical failure criteria attempt to apply the mechanistic principles of linear elastic fracture mechanics to composites by assuming the existence of an "intense energy" region at the crack tip. In practice, however, because of unknown crack dimensions they are two-parameter empirical correlations of test data much the same as the phenomenological criteria. These models were developed for polymer matrix composites where the assumption of an "intense energy" region may be acceptable, but is questionable for ceramic matrix composites. To apply these criteria certain characteristic parameters must be determined experimentally. These parameters are dependent upon the laminate configuration and material system. Like the phenomenological models, the semi-empirical criteria do not describe the mechanisms of failure.

The shear-lag failure criteria are mechanistic models that describe the behavior of the composite at the micromechanics level. The first models did not consider failure in a composite but simply the stress concentration due to broken fibers in adjacent unbroken fibers. Goree and Gross were the first to consider failure. The model included longitudinal yielding of the matrix and matrix splitting, two failure mechanisms seen in polymer matrix composites. Shear-lag models are not currently applicable to ceramic matrix composites. They assume that the tensile load is carried solely by the fibers, while actually, a ceramic matrix has significant load carrying capability.

The fracture mechanics failure criteria are also mechanistic models. The ACK model, based on energy principles, was originally developed for concrete and steel reinforcing wires, but is also applicable to ceramic matrix composites. The multiple matrix fracture and enhanced matrix cracking strain predicted by ACK is seen in many brittle matrix composites. The more rigorous energy analysis of BHE was developed specifically for brittle matrix composites for the cases of frictionally constrained and initially bonded, debonding matrices. The model of primary interest, however, is that of MCE. It considers frictionally constrained, slipping fibers which result in the desired "graceful" failure of the ceramic matrix composite. MCE also consider the transition from notch insensitive large crack behavior to notch sensitive short crack response. Failure due to transverse or compressive loads has not been addressed here. Fracture mechanics models are suitable only for tension loaded ceramic matrix composites. In addition, fracture mechanics models are based on the assumption of a constant interfacial shear strength. The validity of that assumption is being addressed by others.

The criteria discussed so far have all been deterministic. Some phenomenological and mechanistic models can also be expressed in statistical form. There is a great deal of intrinsic variability in the strength of each of the brittle constituents of a ceramic matrix composite but, depending on the composite system, the matrix cracking strength may be deterministic or probabilistic. The ultimate unidirectional composite strength, however, will always be probabilistic since its value is determined by the brittle fiber strength distribution. Thus, statistical models are required for those composite systems which possess a great deal of strength scatter or have linear stress-strain curves all the way to fracture.

The criteria of Wetherhold, Cassenti, and Sun and Yamada are purely statistical and are based on the distribution of composite strength data. Phenomenological models are used as criteria to predict probability of failure of the composite. The three models, described by Jayatilaka, for the ultimate unidirectional strength of a composite are also purely statistical, but are based on the variation in strength of only the composite fiber. In high fiber volume fraction composites, the most conservative and accurate ultimate, longitudinal strength is a function of the bundle strength, provided that the fiber volume fraction and fiber strength are adequate to carry the load after matrix failure. The other two models are less conservative and more applicable to the behavior of polymer matrix composites.

A better approach would be to develop statistical failure criteria based on mechanistic models, as done for monolithic ceramics. These criteria could then be used to extrapolate beyond the range of observations, not on the basis that the distributions they produce can be fitted to existing test data, but that they are germane to the phenomena. These criteria currently do not exist. The most promising model to describe the variability in strength and the mechanisms of failure in ceramic matrix composites would be a statistical failure criteria based on the fracture mechanics models of ACK, BHE and more specifically of MCE.

Fracture mechanics and statistical models, and combinations of these as required by the selected material system, will be incorporated in the near future into an integrated composite design program for component analysis.

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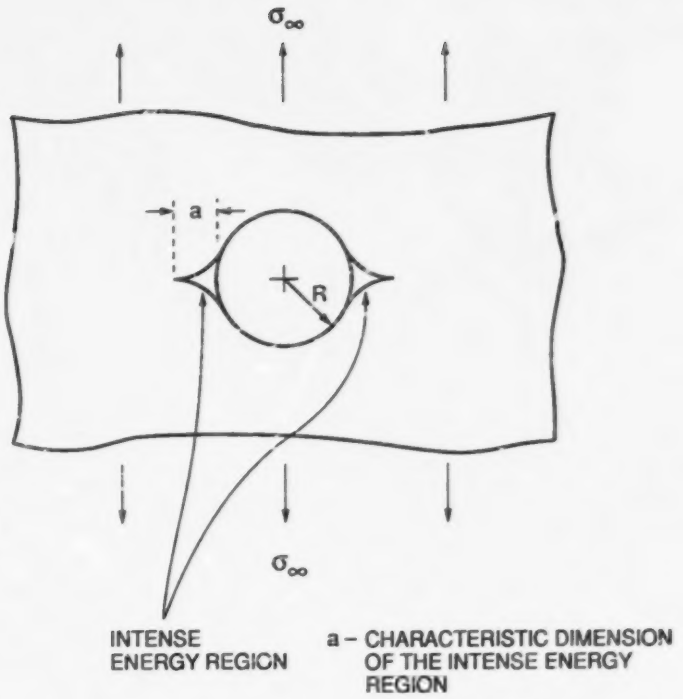


Figure 1. - Semi-empirical model for circular hole showing assumed intense energy region.

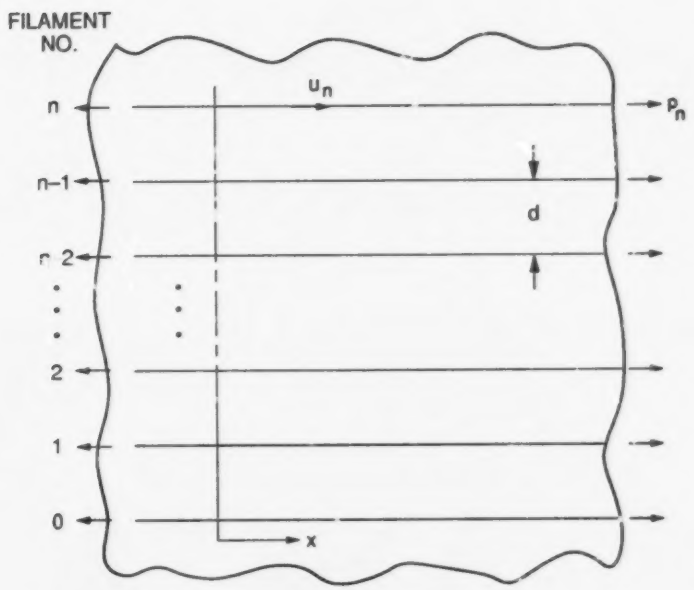


Figure 2. – Forces and displacements in filaments in Hedgepeth's shear-lag model.

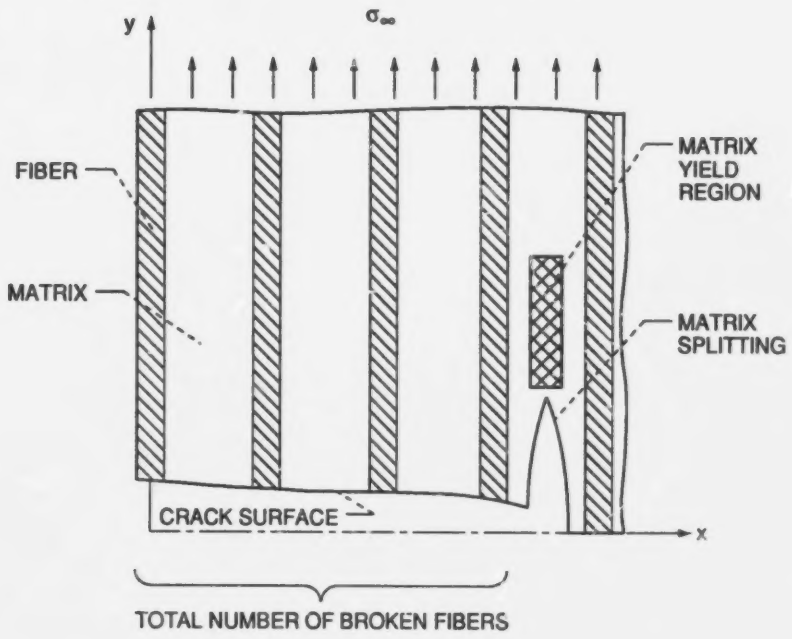


Figure 3. - Goree and Gross's shear-lag model showing matrix damage in the form of matrix yielding and matrix splitting

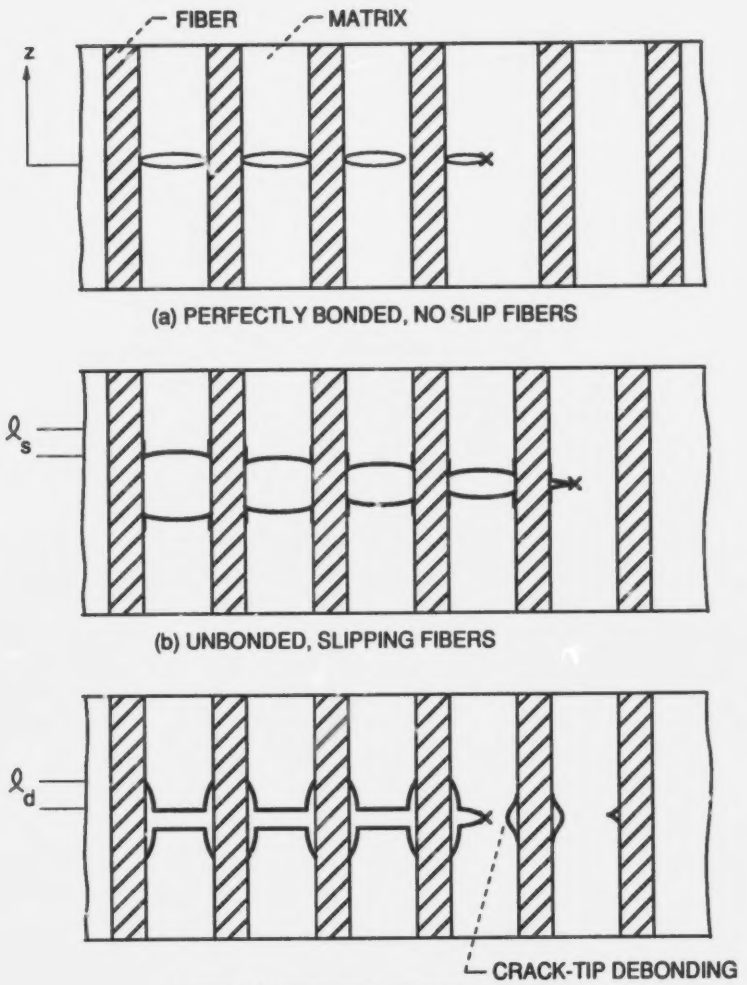


Figure 4. - Various assumed interface conditions for steady-state matrix cracking

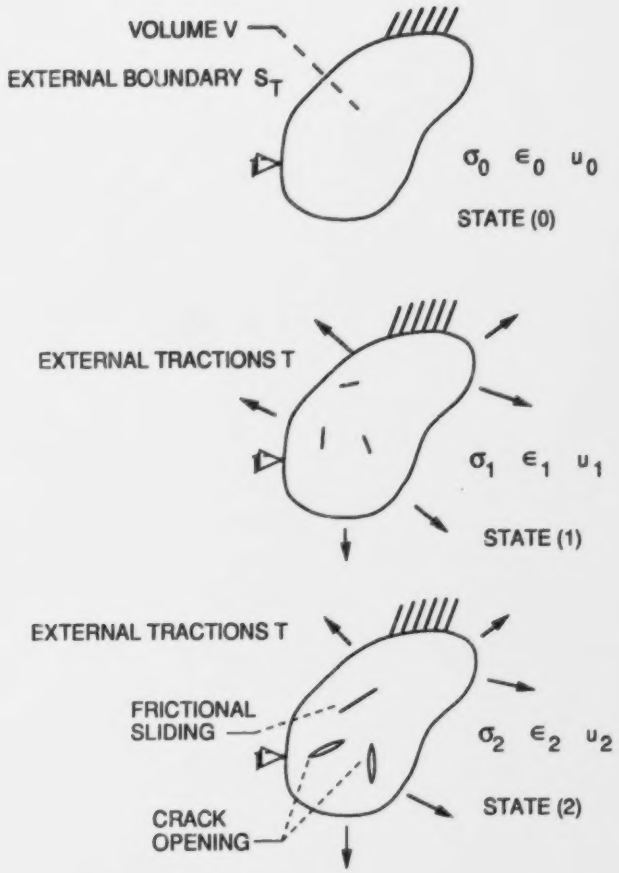


Figure 5. - Three states of energy in deriving Budiansky, Hutchinson and Evan's fracture model.

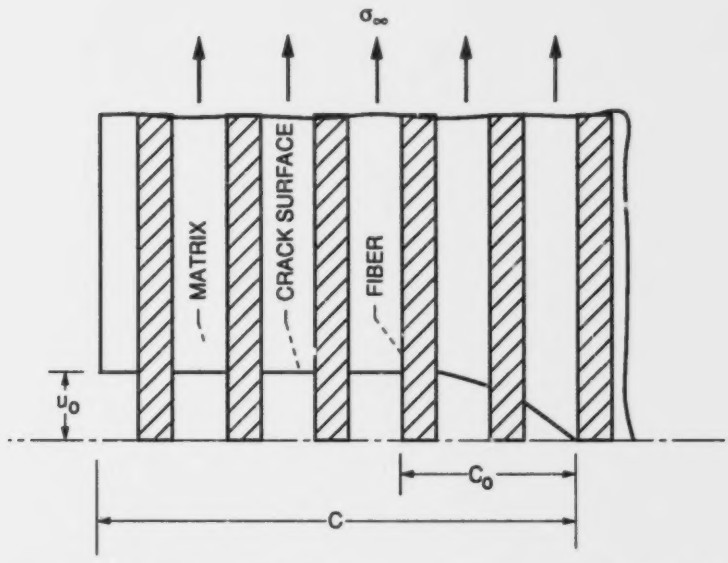
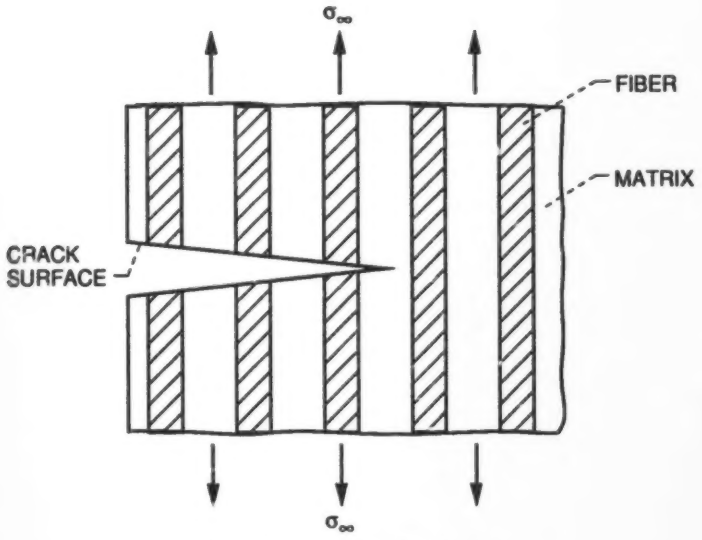
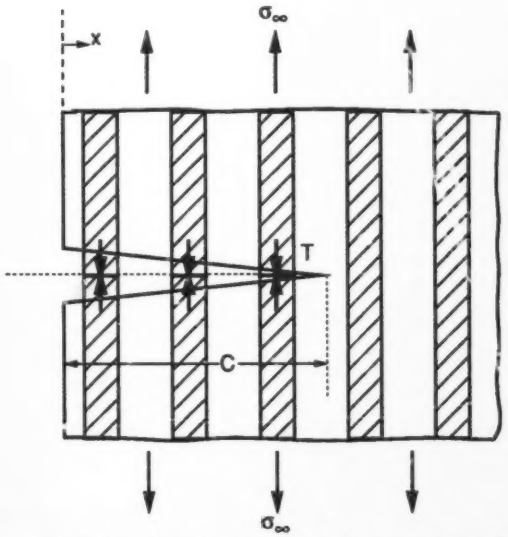


Figure 6. - Steady state matrix cracking assumed by Marshall, Cox, and Evans

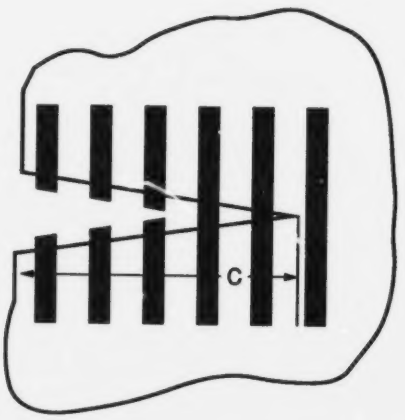


(a) Initially cracked fiber and matrix.

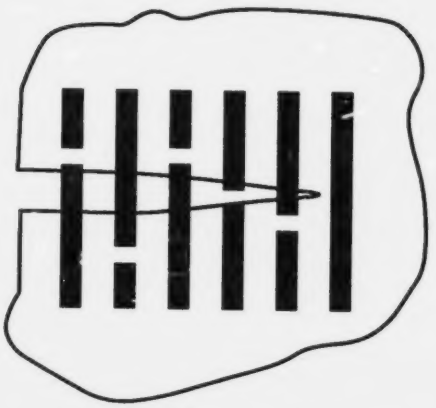


(b) Fiber and matrix after applying traction forces, T.

Figure 7. - Expected crack geometry before and after application of fiber forces to evaluate closure pressure in stress intensity analysis.



(a) Single valued fiber strength.



(b) Statistical variation in fiber strength.

Figure 8. - Crack configurations analyzed assuming fiber breaks in the wake of the crack tip.

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16. Abstract

Ceramic matrix composites offer significant potential for improving the performance of turbine engines. In order to achieve their potential, however, improvements in design methodology are needed. In the past most components using structural ceramic matrix composites were designed by "trial and error" since the emphasis of feasibility demonstration minimized the development of mathematical models. To understand the key parameters controlling response and the mechanics of failure, the development of structural failure models is required. A review of short term failure models with potential for ceramic matrix composite laminates under monotonic loads is presented. Phenomenological, semi-empirical, shear-lag, fracture mechanics, damage mechanics, and statistical models for the fast fracture analysis of continuous fiber unidirectional ceramic matrix composites under monotonic loads are surveyed.

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